Two models for predicting the violin timbre from the material properties of the top and back plates

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Since the pioneering article by Schelleng, the idea of using wood with high E-module and low density has been generally accepted as being acoustically beneficial to musical instruments. We get more sound that way [1]. But how do the properties of wood influence the balance in the sound spectrum from the instruments? How does geometry of the violin and the thickness of the plates influence the spectrum? Will violins made of beautiful wood also sound good? How can we decide what are the beneficial choices with wood and building strategy at an early stage. Are there proper ways to compensate for different wood characteristics and geometry that will lead to the same good (or bad) result? How does the choice of wood properties influence the quality of a bowed musical instrument? Does it exist good and bad wood really?

A basic discussion has also been around the value of studying free plate properties and their significance in the assembled violin. Profiled makers like Schleske has by experiments made on a single instrument shown that free plate tuning does not seem to change the frequencies of the assembled violin in a predictable manner [11]. He studied changes in frequencies and not the levels. Earlier Meinel has done such studies on the same violin [10]. He also studied the influence from arc-height, thickness, varnish and the sound post on the violin spectrum by comparing two or three instruments [10]. He report the frequency spectrum from each played violin tone, probably also during the working steps of thinning a violin. He report that the fundamental grows stronger with thinning of the top plate and that a too thin top makes the sound hollow.

Jansson has also done numerous studies of the different parts and parameters of the violin may influence the admittance levels. Jansson state that a too thin top will lead to a too low “body hill” frequency [private communication]. There should also be a trade off between thickness and radiation efficiency. The question is how and where to stop in the thinning process?

Most makers do have general ideas about this from years of experience, some may even experiment more or less systematically. It takes probably tenths of years of building instruments to achieve a sufficient intuitive model for making still better instruments. And the results might be good from entirely different ideas. The model might even be quite wrong, but the practical result, the finished instrument, may still be good. The complexity is high, the ideas are many, and the time needed to establish a reliable model, intuitive or not, is large. Achieving good tone its also a matter of competition, some may not want to share too much information.

The technically minded reader may know that modelling plucked- or bowed musical instrument sound and timbre may be done by heavy numerical calculation methods like FEM, BEM or other, either in frequency- or time domain (or both) [2]. Such calculations may give answers to some of the stated questions, but heavy models takes a lot of time to build and must yet be done by experts with a solid founding, typically at a University. In the future we may be able to do so on our own PC, so we may have something to look forward to.

Nonetheless I would like to address how the natural variations in material numbers like the density, the stiffness (E-module), damping, thickness and geometry might influence the resulting sound level and timbre from a musical instrument. Two rather simple models is presented; One semi-empirical low frequency resonance level model (RLM) and one finite thin plate frequency average radiation model (FARM) based on theory from the architectural acoustics literature.

Predicting signature modes from free plate levels and frequencies

J. A. Moral carefully accomplished experiments with three tops, three backs, three ribs (and three necks) of different stiffness combined to sixteen violins [4, 26]. Each of the tops, backs and ribs where “stiff”, “normal” and “pliant”. Their free plate 5th mode frequencies (and weights) were 416Hz (91g), 380 Hz (82g) and 316 Hz (68g) for the tops, and 415 Hz (147g), 370 Hz (122g) and 311 Hz (115g) respectively for the backs. The 2nd and 5th mode frequencies of the plates was tuned in octaves. The resistant ribs were 1,5 mm in upper bouts and 2mm in lower bouts, normal were 1mm thick and the pliant ribs were 0,6 mm thick. The material data was not reported.

The experiments gave regression formulas for the admittance level and frequencies of signature resonances for violins based on measured frequencies and admittance levels of the free plates. Six possibilities were chosen to study the effect of the top plates, six with variations only in the back plate and three with only rib variations.

Figure 1 show a sequence of the first signature modes as they usually are found in a violin. The prediction is based on mode #5 free plate levels and frequencies of top and back plates, as seen in Figure 2, and different thickness of the ribs. The plates were made and bi-octave tuned under the supervision by C. M Hutchins, see figure 2. The levels were measured with an accelerometer, giving the velocity levels of the plates in a point, the centre of the plates for mode 5 and at the top of the bass bar at the bridge position for the assembled violin.

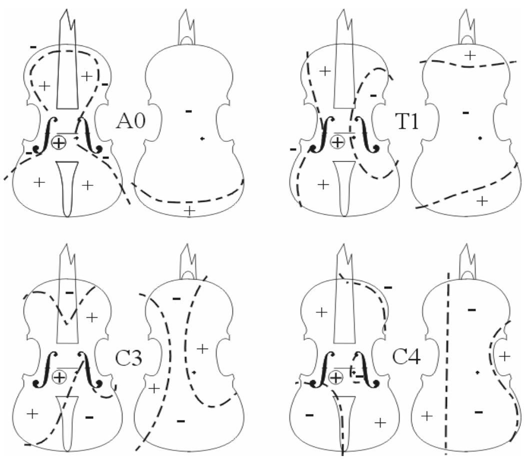
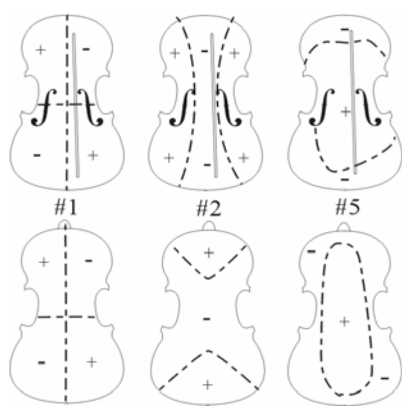


Figure 1: *Signature modes as they are found in a violin. The A0 is the main air "Helmholz" resonance, the T1(B1$^-$) and the C3 (B1$^+$) are the first and second important structural resonance, respectively. Both these are in the area of Saunders "main wood" [9]. The C4 is what Rodgers called "ring mode in the back". The A0, T1, C3 and the C4 are the most significant radiators below ~800 Hz, the latter somewhat less than the other three [9].*

Figure 2: *Free plate modes many makers use for a control of the relation between longitudinal, cross grain and flexural stiffness and mass distribution of violin top- and back plates. In octave plate tuning the mode #2 and #5 of the top and mode #2 and #5 in the back are tuned in whole octaves. The modes in the top- and back plates are tuned to the same frequency, usually around 350-380 Hz for the #5 mode. FEM calculations has shown that the #1 is somewhat more influenced by the G, the #2 by the $E_\perp$ and the #5 by the $E_{||}$ than the other wood elastic parameters. The plates in Morals experiment were octave tuned [4].*



Morals regression data has been tested for significance. The following regression formulas are shown with bold text for the relations that are significant on the 5% level, and with normal text for relations significant on the 32% level. We may read the 5% level results as "significant" and 32 % levels as being "interesting". The regression is based on only six points. A larger number of test samples would have been good, but would require a tremendous work to accomplish. It is, however, the largest known published set of data in the working processes of violins. Other works has been on the same violin with different working steps, like the experiments by Meinel and Schleske [10, 11].

$\Delta L_{A0} = \mathbf{1.30{*}\Delta L_{T5} - 0.31{*}\Delta L_{B5}} + 0.00{*}\Delta t$
$\Delta L_{T1} = \mathbf{0.67{*}\Delta L_{T5}}$
$\Delta L_{C3} = \mathbf{0.47{*}\Delta L_{T5}} - 0.41{*}\Delta L_{B5} - 0.01{*}\Delta t$
$\Delta L_{C4} = \mathbf{0.17{*}\Delta L_{T5} + 1.10{*}\Delta L_{B5} - 0.02{*}\Delta t}$

$\Delta f_{A0} = \mathbf{0.10{*}\Delta f_{T5}} - 0.05{*}\Delta f_{B5}$
$\Delta f_{T1} = \mathbf{0.58{*}\Delta f_{T5}} \qquad \mathbf{- 0.05{*}\Delta t}$
$\Delta f_{C3} = \mathbf{0.23{*}\Delta f_{T5}} - 0.18{*}\Delta f_{B5} + \mathbf{0.01{*}\Delta t}$
$\Delta f_{C4} = 0.42{*}\Delta f_{T5} + \mathbf{0.39{*}\Delta f_{B5}} + 0.03{*}\Delta t$

Where the ΔL_i is the change in vibration admittance level [dB re 4 s/kg] for resonance i = A0, T1, C3 or C4 in the violin body from a change in the free plate levels ΔL_j [dB re 4 s/kg] of mode 5 in the top, T5, or the back, B5. (The top plates had mode 1, 2 and 5 an octave apart. The backs had mode 2 and 5 an octave apart.)
Δt is a change in thickness of the ribs in %
Δf_i [Hz] is measured changes in the resonance frequencies of the violin body (A0, T1, C3 and C4) to changes in the free plate 5th mode frequencies, Δf_j.

We see that the top plate seem to be more influential than the back plate for the first two signature modes, the A0 and the T1. The C3 seem to be about equally influenced by both the top and back, while the C4 seem to more influenced by the back plate. The ribs are somewhat less influential, but still significant. One may draw the conclusion that changes of the free plate properties does indeed influence the assembled violin resonant properties.

From the measurements on each of the test violins Moral found that a pliant top, pliant ribs and a normal back gave the highest average T1, C3 and C4 levels and lowest standard deviation between these three resonance levels. These measures was related to the quality points from a violin competition by a regression formula giving highest value for this combination of plates and ribs. This support the earlier used tap tone rule with the back tuned a half tone higher than the top [Hutchins and Saunders first Scientific American article from 1962, and 29].

Average velocity levels of a vibrating structure and the sound pressure levels generated by it is closely correlated [14, 18, 19]. The relation when it comes to details is rather complicated, we will return to that. But to a first approximation we may think of them as just being proportional.

Are the natural variations in free plates levels and frequencies audible in a finished violin?

From measurements by Jansson and Niewzcyk [8,9] we know that average top and back plate levels of mode 5 is around **–** 18 dB and that the natural variation, two standard deviations, 2 * *s*, including 95 % of the data set, is about ± 6 dB. Knowing that we may just notice a difference in sound level of about 0,5 – 1 dB, the natural variation in free plate tap tone levels of ± 6 dB should be clearly audible. The free plate levels has surprisingly not been given much attention in the literature maybe caused by practical difficulties in measuring the levels.
The natural variation in the signature mode levels can be seen from the shown level regression formulas. The ± 6 dB variation in the free top plates is expected to give a ± 8 dB (± 1.8 dB for the back) variation in the A0 admittance level, a ± 4 dB variation in the T1 level, a ± 3 dB (± 2.4 dB) variation for in the C3 level, and about ± 1.2 dB (± 6.6 dB) for the C4 resonance. Such variation in the finished violin is likely to be clearly audible. These changes are likely to influence the fundamentals of much of the played low and mid tones on a violin. We may understand that such variation will influence the perceived quality of violin sound. This is also in agreement with the practical experience many makers have.

Changes of rib thickness has the unit of percent. A 100 % change of rib thickness is expected to give a change of the C4 level by about 2 dB and the C3 level about 1 dB, both effects are small but, still, audible.

We do probably not detect precise slight changes in the signature mode frequencies while playing a violin. A small change will however easily be detected in the tapped free plates. To notice a frequency change while playing, it should at lest be moved about a semitone, about 30 Hz. To achieve such a change in a violin the free plate frequencies should change one or two whole tones (60 - 110Hz), which is quite dramatic in terms of free plate tuning. However, a whole tone change of free plate #5 frequency will, of course, change the admittance levels of the free plate and thus levels of the signature modes in the competed violin. An interesting question is how much and how does it influence the timbre?

Modelling mode average levels of the free plates

To calculate the level changes from changes in material properties, the thickness and the geometry we need a model. Theory for a thin homogenous flat plate give the following expression for the surface mean admittance level [dB re 4 s/kg] [based on 14]:

$$L_v = 10\log\left[\frac{1}{4}\left(2.3\sqrt{E}\rho^{3/2}h^3 S\eta 2\pi f\right)^{-1/2}\right]$$ (Not quite sure here if it should be 20*log[ ] or 10*log[ ])

Which may be rewritten in the form:

$$L_v = 11.8 - 2.5\log(E) - 7.5\log(\rho) - 15\log(h) - 5\log(S) - 5\log(\eta) - 5\log(f)$$

Where:
E is youngs modulus [MPa], typically 14000 MPa for spruce, and 10000 MPa for maple
ρ is the density [kg/m^3], typical values are 440 kg/m^3 and 660 kg/m^3 for spruce and maple respectively
h is the thickness of the plate [m], here a good guess is 2.6mm for the top and 3.0mm for the back plate
S is the surface area of the plate [m^2], the area of a violin plate is here calculated as 0.18 * 0.33 m^2
η is the damping constant (=1/Q) [1], typically 0.015 (Q = 67), same for both plates
f is the frequency [Hz], typically 380 Hz for the #5 mode, but the model holds for any frequency up to a given limit where thin plate theory no longer is valid.

Using the typical values for the input parameters from wood data and an average thickness of 3.1 mm (rather high) we get L_{Top} = - 17 and L_{Back} = - 18 dB respectively, close to the measured average of - 18 dB for the 5th mode for both plates [9]. This is not bad when we know that the simplified model is for a homogeneous flat plate, neglecting the ortothrophy of wood. The increased stiffness from the bass bar and the arching, would reduce the level of the top plate a bit, but the reduced cross grain stiffness ($E_\perp$ - module) of wood would probably made the levels slightly higher.

It is interesting to note that the measured #5 plate level give a close estimate for the violin plate frequency average response around that frequency. It support the idea that the level of that mode is an important measure when dealing with free plates.

We see that the simple model contain several of the most important wooden parameters, thickness, geometry and the damping. We should now be able to see that the thickness and the density are likely to be the more influential parameters for the average velocity level of the plates and thus for the signature modes in the violin. We are also able to see how the effect of varnishing will influence the levels of the plates. Varnish increase damping, density and thickness at the same time, all effects giving lower levels.

We are able to see the effect a change in the material parameters and thickness of the plates will have on the signature modes:

- Reducing top plate thickness will increase the vibration levels of the signature modes controlling the low- and mid frequency response of a violin.
- Using lower density wood in top plate will increase the levels of the signature modes.
- Using top plate wood with high stiffness will reduce the vibration levels of the signature modes slightly.
- Reducing back plate thickness will reduce the signature modes A0 and C3 levels but will increase the level of the C4.
- Using lower density wood in the back plate will reduce the level of the A0 and C3 and increase the C4 level.
- Using back plate wood with high stiffness will increase the A0 and C3 levels slightly and reduce the C4 level slightly.

Modelling frequencies of the free plates and the signature modes, RLM

A complete model for the level and frequencies of the signature modes would be possible when we have a proper model for predicting the frequencies. Lately an interesting regression model for the calculation of frequencies of real violin top plates has been proposed by H. Meyer [6]. We need a model for the back plates, however, and the idea was to use the input data for the rectangular plate from the level calculations.

McIntyre and Woodhouse [21] has presented formulas to use for measuring wood properties of rectangular wooden (ortothropic) plates based on a given geometry and the frequencies of low frequency modes. A rewriting of the formulas given in [21] for resonance frequencies for these three modes from input of along grain and across grain E modules, flexural G-module, density, thickness and dimensions of the flat plate.

We know that flat plate frequencies are lower than what we find for curved plates. Schumacher [7] used FEM calculations on a double curved violin like plate with changing arch height to track the frequency changes from a flat to a curved plate. Based on these calculations the flat plate mode frequencies are multiplied with constants close to the results for a 20° arch from his work [7]. These are without a bass bar and f-holes, so constants were best fitted to the free plate frequencies of the three top and back plates of the Moral experiment. (I ended up with 1,3 times the flat top plate frequencies for mode 1 and 2, and 2.3 times for the ring mode 5. For the back the numbers are 1, 1 and 2.2 respectively).

The predicted frequencies work well when operating close to normal thickness (2.3 – 3.1 mm) but tend to be a bit too high for thick plates and a bit too low for thinner plates. This may be an effect of choosing a bit too small rectangular geometry to fit the radiating surface area also giving an useful, but rough, prediction of the weight of the plates. Keeping it in mind that the predicted frequencies of the signature modes is rather insensitive to changes in the free plates, and that the changes probably must be rather large to be of importance, the model should be sufficient for the purpose when we stay close to normal plate thickness.

The frequencies, levels, Q-factors, and weights of the plates were given, unfortunately the thickness data and densities were not. The fit was thus based on the assumption of having plates made from average wooden data gathered from Haines [5]. That led to the best fit for the weight and resonance frequency with a 3.1 mm thickness for the top plate with normal stiffness ($f_{\#5}$ = 380 Hz, weight 82 g), 2.6 mm for the pliant plate ($f_{\#5}$ = 318 Hz, 68 g) and 3.5mm for the stiffest plate ($f_{\#5}$ = 417 Hz, 91 g) Here the weight of the bass bar is included in the thickness, this gives a bit too large thickness for the top plate, in the order of 0.2 mm for a 6 g bass bar.

For the back plate the same type of fit gave 3.1 mm for the normal plate ($f_{\#5}$ = 368 Hz, weight 122 g), 2.9mm for the pliant plate ($f_{\#5}$ = 308 Hz, weight 115 g) and 3.7mm for the stiff plate ($f_{\#5}$ = 413 Hz, weight 147 g)

(Plan to add a figure or table showing a comparison between the measured and calculated levels and frequencies. The data from Morals work and possibly the other. Anything that underlies the conclusions!)

Test of the RL Model on data from the literature

Tests on thinning down of violins and measuring the resonant properties for each working step has been reported by Meinel, Schleske and Jansson and Niewczyk [10,11,8]. The experiments by Meinel was reported in eight working steps with the thickness data and some examples of the response changes. For the Schleske data there was weights, frequencies and thickness of the free plates given, so an estimate for the density of the wood was possible. In the Jansson and Niewczyk experiments, the E modules, G modules, densities and thickness (although quite coarse) was reported.

Table 1: *Input data to the model. Average material data from Haines [5] was used when data was not reported.*

Experiment	**ρ [Kg/m^3]**	**$E_{\|\|}$ [Gpa]**	**$E_\perp$ [Gpa]**	**G [Gpa]**	**H [mm]**	**Comment**
Meinel top/back	440/660	14/10	0.83/2.18	0.79/1.95	4.8-2.3/5.5-2.97	ρ, $E_{\|\|}$, $E_\perp$ G assumptions
Schleske top/back	401/566	14/10	0.83/2.18	0.79/1.95	4-2.7/5-2.75	ρ fitted, $E_{\|\|}$, $E_\perp$ G assumptions
N&J top 70/back 67	414/554	12.4/13.9	0.79/1.56	0.7/1.44	3.3/3.3	H assumption
N&J top 57/back 61	377/562	13/12.2	0.85/1.45	0.57/1.31	3.8-3.3/3.3	H assumption
N&J top 67	363	6.8	0.72	0.7	3.8	H assumption
Buen top/back	430/600	14/9	1/1.5	0.79/1.95	2.9-2.3/3.4-3.2	ρ, $E_{\|\|}$, $E_\perp$ G assumptions

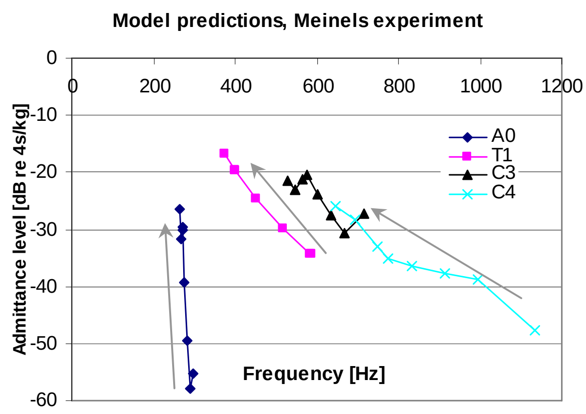


Figure 3: *Meinels experiment had eight working steps. From the 6th and up the sound of the violin was good. The arrows indicate the direction of change with thinning of the plates. Predicted changes of frequencies are larger than measured.*

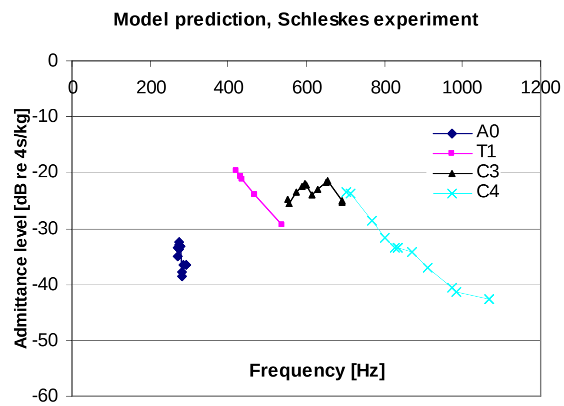


Figure 4: *Schleskes experiment had fourteen working steps. From the 10th step the sound was good. His back is thinner in the end stage giving a higher C4 and the top is thicker giving a lower C3 and A0 levels than in the Meinel experiment. Changes in frequencies are larger than measured.*

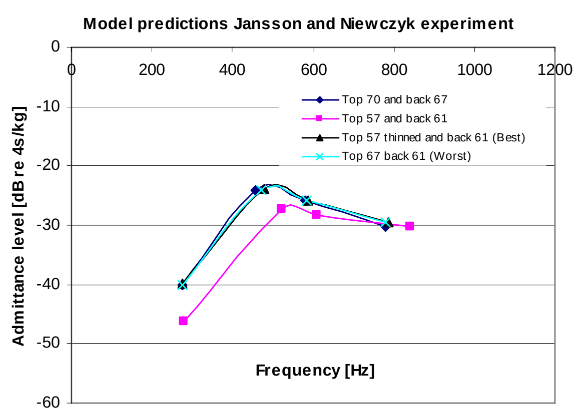


Figure 5: *Calculated levels and frequncies of signature modes for the Jansson and Niewzcyk experiment. The top plates were thicker than normal, but still it is interesting to note that the levels are similar for two instruments that were regarded as good and not so good. Their critical frequencies, f_c are 2880Hz (Best) and 3400 Hz (worst) respectively. The violin with the lowest admittance has a f_c = 2500 Hz. Thickness values were coarsely given though.*

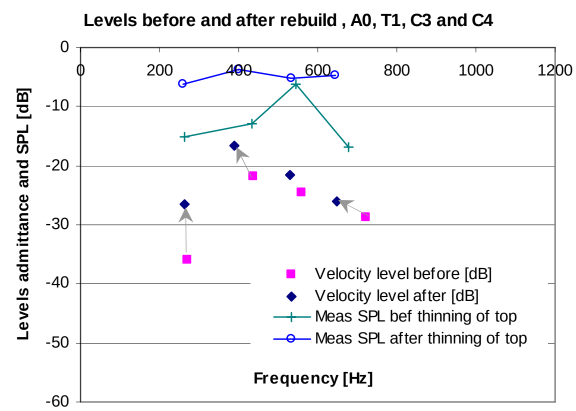


Figure 6: *Measured SPL and calculated admittance levels for a violin before and after thinning of the top plate from 2.9 mm to 2.4 mm and back plate from 3.4 mm to 3.2 mm. It has predicted levels similar to Meinels end stage in Figure 3. The measured levels change more than predicted for three of the modes. This may be caused by the reduced bass bar length (7 cm), about 6 mm in height and about 3 g in weight. Total weight reduction of the top was about 10 g.*

The level changes are likely to be more correctly modelled than the frequencies. The predicted level changes in figures 1-4 will be clearly audible. The model does indeed support the empirical findings by many makers, including Meinel [10], that thinning the violin top plate will give higher low frequency response in a violin.

Moral and Jansson measured the quality ranked violins in a competition and found a regression formula for the quality points from signature resonance levels: QP = 0.6 * (Average level of T1, C3 and C4) + 0.4 * Standard deviation of the levels of the same plus a constant. QP = (6*(0.6 Avg L + 0.4*Sd L) + 27) The original work also contain a factor from the high frequencies and from the balance between the two sides of the bridge, the equality of the levels between G-string and E-string sides of the bridge [4]. This relation may be used for prediction of the sound quality as compared to the violin competition in Sweden the regression is based on. The winner in that competition had QP ~ 75 and a concert violin made by Andrea Guarnerius had QP ~ 80.

Table 2: *Calculated sound pressure p/F for each signature mode, quality points and critical frequencies of the top plates of the violins.*

Experiment work stage	**L(A0) [dB]**	**L(T1) [dB]**	**L(C3) [dB]**	**L(C4) [dB]**	**QP [1]**	**f_c Top/ Back [Hz]**	**Comment**
Meinel start	-31	-23	-17	-40	6	2064 / 2611	
Schleske start	-22	-18	-14	-35	24	2365 / 2660	
Meinel, „good“	-6	-8	-10	-25	56	3811 / 3850	
Schleske, „good“	-8	-9	-11	-26	54	3378 / 3427	
Meinel, end	-2	-5	-11	-18	74	4308 / 4835	
Schleske, end	-9	-8	-14	-16	74	3504 / 4836	
N&J top 70/back 67	-16	-12	-15	-23	58	3087 / 3381	
N&J top 57/back 61	-22	-15	-17	-23	55	2505 / 3635	
N&J top 57 thinned/back 61	-15	-12	-15	-22	60	2884 / 3635	Best violin
N&J top 67/back61	-16	-12	-15	-22	60	3398 / 3635	Worst violin
Buen, before	-11	-10	-13	-23	60	3265 / 4245	
Buen, after thinning	-1	-5	-10	-20	71	4053 / 4510	Good low frequency response

We need a model for the behaviour at high frequencies as well. Studies by Bissinger has shown that the radiation ratio and the critical frequency is an important factor for the quality of a violin [22]. These values (radiation efficiency and the critical frequency) has been used extensively in building acoustics literature and engineering for at least 50 years, e.g. for the prediction of sound transmission coefficients (STC) of light weight wall partitions, or practically utilised by covering well radiating thick heavy wall partitions with low radiating thin plates of high stiffness.

A frequency average radiation model including high frequencies, FARM

There exist estimation procedures for calculation of sound power levels from machines, wall partitions or other radiating structures, based on their measured or calculated average velocity response over its surface. One such relation is given by Takatsubo et al 1983, [rewritten in 15]:

$$L_w = 10\log\langle v^2\rangle + 10\log S + 10\log\sigma + 146 \qquad \left[dB \quad re10^{-12}W\right]$$

Where S is the surface area [m^2], $<v^2>$ is the mean square normal surface velocity [m/s] averaged in time and over the radiating surface. The quantity σ is a measure of the efficiency of radiation, the radiation ratio [1], since it can be larger than unity. Generally the radiation ratio is unity or less.

We may look at the violin plates as being baffled by the ribs and the plate on the opposite side. There is probably a slight leakage through the f-holes, but above the air resonance and not close to the air resonances this contribution is likely to be small.

The mean square surface velocity averaged over time, $<v^2>$, can be estimated from the earlier given equation for the velocity level and the relation for the mobility; $L_v = 10\log\left[\frac{\langle v\rangle}{4\langle F\rangle}\right]$ [dB re 4 s/kg]

There exist a number of equations for estimating the radiation ratio. They have the properties common that increased plate thickness, and reduced plate periphery will increase the radiation ratio. Also stiffeners at one side of the plate or a cylinder will increase the radiation ratio [Fahy]. This ratio give an comparison between the radiation from a circular baffled piston (in an infinite wall) vibrating at the same average velocity level as the radiating structure we study, and the radiation from our radiating structure.
The most important identity for the radiation ratio, σ, is the critical frequency, f_c, where the radiation ratio is unity or more. The critical frequency for a homogenous flat plate is given by Vigran [14]:

$$f_c = \frac{c_0^2}{1.8 c_L h} = \frac{c_0^2 \sqrt{\rho(1-\upsilon)}}{1.8\sqrt{E}h}$$

where c_0 is the speed of sound in air (342 m/s @ 20 °C), c_L is the longitudinal phase speed of sound in the material [m/s] and ν is the Poisson ratio [1] typically ~ 0.5 for spruce wood.

This expression show that f_c will decrease with lower density, ρ, of the wood, by higher stiffness, E, and higher thickness h of the plate. For those with an ultrasonic sound meter, like the "Lucci meter", it would be interesting to compare measurements of C_L and h of good and not so good violins. Both measurements may easily be accomplished in a violin maker workshop. If Bissingers findings prove to be correct, we would expect that better instruments have a lower f_c and possibly a higher C_L..

For frequencies higher than f_c the radiation ratio has a 1/√f dependence, for frequencies just below the critical frequency the ratio goes down close to a quadratic term, and almost linear at lower frequencies.

One such equation for the average radiation ratio for a thin rectangular homogeneous plate is given by Leppington et al 1982 [Vigran or original work]:

$$\sigma = \frac{Uc_0}{2\pi^2\sqrt{f \cdot f_c}S\sqrt{\chi^2-1}}\left[\ln\frac{\chi+1}{\chi-1} + \frac{2\chi}{\chi^2-1}\right] \qquad for \quad f < f_c$$

$$\sigma = \sqrt{\frac{2\pi f}{c_0}}\sqrt{a}\left(0.5 - 0.15\frac{a}{b}\right) \qquad for \quad f \approx f_c$$

$$\sigma = \frac{1}{\sqrt{1-\frac{f_c}{f}}} \qquad for \quad f > f_c$$

a and b are the sides of the rectangle, $a > b$, $U = 2(a+b)$, and the parameter $\chi = \sqrt{\frac{f_c}{f}}$. The equation assumes that the f_c has to be far higher than the lowest resonance frequency of the plate, and that the radiation is from resonant radiation from several modes. For a violin the latest assumption is believed to be valid from about 1 – 1.5 kHz where the modal overlap is considerable.

Test of the FAR Model on data from the literature

The data from table 1 is used for the FARM on the same experimental material as for the RLM.

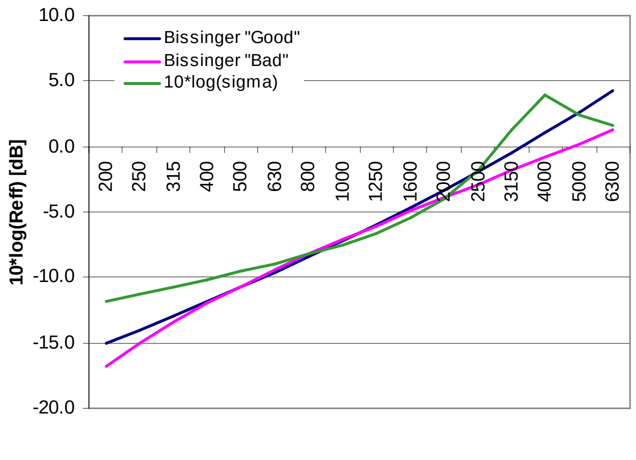


Figure: *Green curve: calculated radiation efficiency for a 2.8 mm thick spruce plate of dimension 0.18*0.33 m^2 , density 420 kg/m^3 and $E_{||}$ = 15 MPa. Critical frequency: 3340 Hz. Blue and pink curves: Measured radiation efficiencies for the group of "good" and "bad" violins [22] The model is as we see "no catastrophe".*

The critical frequency may come to a lower frequency than calculated by the FARM by the bass bar, the sound post and possibly the varnish, all stiffening the plate. A stiffening element like a bar, makes the effective plate periphery longer, an effect that increases the radiation efficiency [18,19].

The top in radiation efficiency as predicted by the Leppington plate model for radiation efficiency does, according to Bissinger [22] appear as a "knee" in the measured curves for the radiation efficiency. The Leppington model is probably made for the building acoustics prediction of STL through wall partitions, being baffled by the flanking walls. A violin is baffled by its own body being "wrapped around itself" and the effectively radiating "corner and edge modes" will appear in a different manner, if at all [24]. It might also be that the violin plate, being much smaller in size than a wall partition, is probably outside the input data set to the experimental determination of the Leppington formula. Also we should expect effects of the ortothrophy of wood, making the radiation efficiency quite complicated, at least "two dimensional" [23].

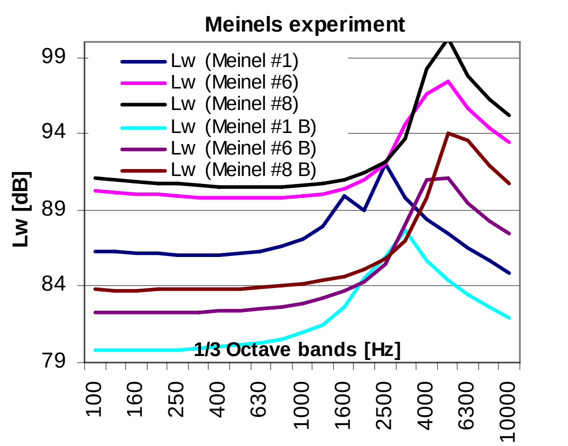


Figure: *Start-, stage from which the violin is "good" and end stage. Splitted top is an artifact of the model(the shift between the three equations for the radiation ratio and the large width of the 1/3 octave bands).*

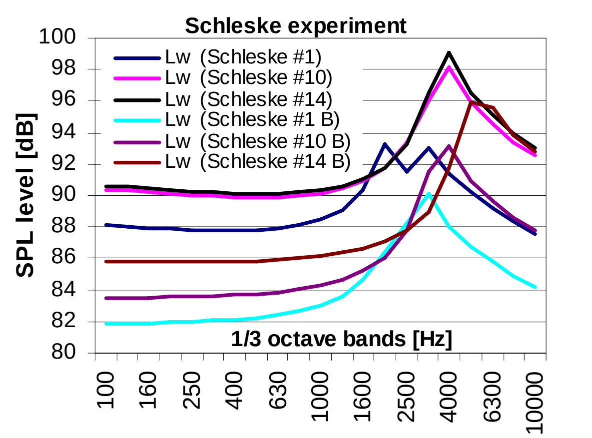


Figure: *Start, stage from which the violin is "good" and end stage. Splitted top is an artifact of the model.*

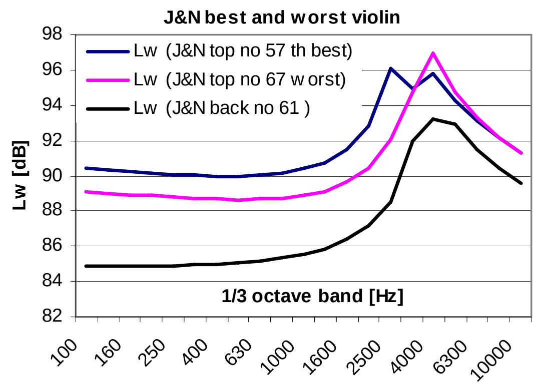


Figure: *J&N experiment best and worst violin, both with the same back. Split in blue curve is an artifact of the modelling.*

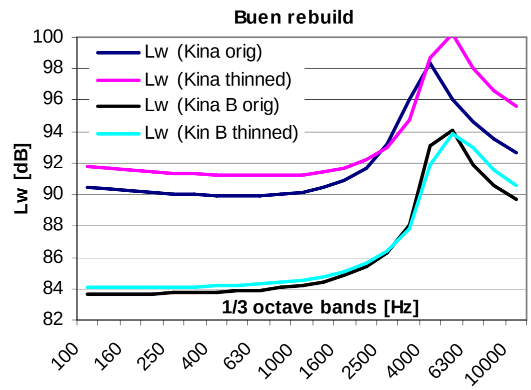


Figure: *FARM calculations of a thinning experiment on a Chinese made hardanger violin.*

(Why is there a difference in J&N low frequency response when it is small in the RLM? Answer: The radiation may be larger than the admittance due to the radiation properties, stiffer plates radiate more due to lower critical frequency. Explain why the low frequency response is less different for the FARM than RLM)

We see that thinning generally increases the sound level in any frequency band, but that the changes are larger beyond the critical frequency of the start stage. Thinning changes the frequency distribution. It is interesting to see that the top plate no 67 in spite of having the lowest density it has a lower radiation due to a low E module. The top plate 57 had a lower critical frequency when it was thicker, but thinning 0.5 cm gave increased low frequency response, (see results from RLM calculations) , and still a lower critical frequency than top plate no 67. The violin with top plate 67 did not have a bridge hill, while the top plat 57 had a bridge hill. Does the critical frequency influence the admittance as well?

Violins are played up to a fundamental around 4000 Hz, and the string harmonics force will sink about 6 dB per octave. The sound spectrum from the violin is therefore dominated by the fundamentals and their two three first harmonics, of course depending on the detailed frequency response of the violin. Measured played violin spectra does not show a high frequency top being much stronger than the low frequency response, see figure x below. This is partly due to the heavier excitation by heavier strings at lower frequencies. Higher tones are also played with strings shortened by the fingers making the force lower, but this is compensated with the bow moved closer to the bridge and the bow speed may be increased. By professional players high frequency notes may be played with a painfully high sound level.

(Show a graph indicating an average force spectrum in 1/3 octave bands from a string)

A violin is a shell structure that may be modelled better with an expression for a closed elliptical cylinder, or other box like structures. Still the flat plate model is likely to explain the more important features connected with the critical frequency and the radiation in general. Other important structural influences like the height, weight geometry and resonant properties of the bridge, the position of the sound post, string angle, plate arching, more detailed instrument geometry, coupling structures is not included in the models. But they are able to predict influences on the material parameters, thickness changes, and influences material properties may have from other influences like humidity, varnish, baking and so on.

Table 3: *Calculated contribution from variations in top and back plate material parameters and thickness on the radiated sound level from violins. Data: @ 500Hz FARM/RLM, @3.15kHz: FARM.*

Parameter	Nominal values	Standard dev. σ	σ Lp [dB] @ 500Hz	σ Lp [dB] @ 500Hz	σ Lp [dB] @ 3150Hz	Comment
H top [mm]	2.6	10 %	-0.6	-2.2	0.4	Frequency dep. Hnom data from [Loen]
H back [mm]	3.1	23 %	-0.6	1.2	0	* First three res. C4 goes down, data [Loen]
ρ top [kg/m3]	440	10 %	-1	-0.6	-1.6	Frequency dep. [5]
E top [N/m2]	14000	14 %	0.2	0.6	0.8	Frequency dep. Based on data from [5]
ρ back [kg/m3]	660	9 %	-0.8	0.2	-1.1	* First three res, C4 goes down, data [5]
E back [N/m2]	10000	16 %	0.2	0.2	0.6	* First three res. C4 goes down, data [5]
η top (1/Q) [1]	$0.37 * f^{-0.39}$	8 %	-0.3	-0.2	-0.3	Finished violins, based on data from [25]
η back (1/Q) [1]	$0.74*f^{-0.49}$	10 %	-0.4	-0.1	-0.4	Finished violins, based on data from [25]
Sum $\sigma_{Lp} = (\sigma_H^2+\sigma_H^2+\sigma_\rho^2+\sigma_E^2+\sigma_\rho^2+\sigma_E^2+\sigma_\eta^2+\sigma_\eta^2)^{0.5}$			1.6	2.7	2.3	95% confidence interval (4*σ) = 6.4-10.8 dB

Table 3 show what effect a one standard deviation change in any parameter is predicted to give in the resulting sound output. Thickness is most important, after that the density of the material and the E-module follows. The models predict a larger influence on the radiated power from the top plate than from the back (we now know that this is probably not entirely true [27]. Our assumption that the admittance and the produced sound pressure levels are proportional in the RL model is probably not a very good one, at least not for the modes influenced by radiation through the f-holes. This radiation is not included in any of our models.).
We also see that the predicted summed variance is from 6,4 to 10,8 dB from natural variations in wood parameters and thickness.

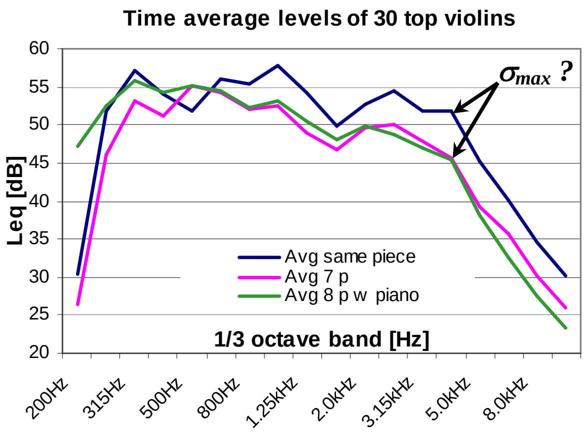


Figure x: *Time average spectra from recordings of 30 top violins (15 Stradivarius and 15 Guarnerius del Gesú`s). Blue curve is the same Sibelius excerpt played on all 30 violins, pink line is 7 different pieces on 14 violins, and the green curve is 8 different pieces on 16 of the violins accompanied by piano. The arrow indicate the possible maximum radiation ratio in the 4 kHz band. (Bandwidth of 4kHz band is 920 Hz)*

The influence on violin tone from changes in the relative humidity in air

Both the density and E-module of wood is dependent on the relative humidity of air. The density increases notably with humidity while the E module will decrease considerable with the humidity. The poisson number is sensitive to humidity, and the E-module is also slightly dependent on temperature.

An interesting question is “Are normal changes in wood RH audible?” Many professional musicians prefer to keep their instruments in a relatively humid environment, the wood does not crack so easily that way. But how does it affect the acoustics of the instrument?

The equilibrium humidity of wood is dependent on the RH of the surrounding air and slightly on the temperature. Around room temperature wood humidity is about 6 % at air RH of 30 %, and 11 % at RH of 60 %, that is about a doubling of the water content in the wood for a doubling of RH in air. In a wooden piece of 400 kg/m3, the amount of water will be about 24 kg/m^3, while at 11 % it will be about 44 kg/m^3. The density have thus increased 5 % to 420 kg/m^3. This change in density alone will give about -0.5 dB change of level at lower frequencies and about -0.8 dB at high frequencies.

The effect will depend on each instrument thickness, especially the effect at higher frequencies. One way to test this is by making recordings of violins at low and high relative humidity in a small room.

The E-module in the tangential and radial direction is almost linearly dependent on humidity content below 30 %. For the longitudinal component, the relation is somewhat weaker. A 10% lower E-module would, however, increase the effect on loudness, giving some additional –0.2 dB and –0.8 dB levels respectively. The critical frequency will go up, the balance of the average spectrum will favour the lower frequency region by some 0.9 dB, the instrument is likely to sound less bright and we may expect that the effect will be just about audible.

Increased humidity will also reduce the air damping of higher frequencies in a larger auditorium, making the sound in the room become more brilliant. The effect is largest in the region RH > 10 % to about 30 % RH, such low RH's are common in dry Nordic winter climate. For a listener in a concert hall this effect (with increasing RH) will reduce the wood effect of RH making darker sounding violins sound slightly more in the brilliant direction.

Variation of the air humidity will change the frequencies of the violin plates we wish to tune. The levels will also change, in many cases more than a finished violin is likely to do, see the RLM. A good idea would therefore be to use tuned wooden bars as tuning aid guides for tuning violin plates. They should follow the changes of the plates somewhat with changes in RH. The bars should be of a type close to the top and back plate wood of good quality. This method has been used as a tuning aid by makers in my own family.

Ideas that might be included:

- A figure showing the changes in spectra with natural variation in wood parameters
- A figure showing the $10*\log(\langle v^2 \rangle)$ and $10*\log(\sigma)$ showing the trade off between thickness and radiation ratio.
- How much does the sound level change with a change in plate surface area? For table on influence.
- More information in the introduction about the present knowledge on the mechanical details about bridge resonace (Beldie CASJ Nov 2003), body hill (Jansson), Thoughts around the body hill by Woodhouse (SMAC 03 proceedings and CASJ article in Janssons 60-year anniversary issue). Will the critical frequency appear in an admittance measurement? Answer: - No, but will by the reciprocal experiment! (An accelerometer on the body of the violin and exiting the violin by a loudspeaker, thus activating the radiation pick up)
- Measure the critical frequency by white noise broad band excitation in a reverberation room.

Conclusion

The resonant low frequency response for violins may be calculated by a hybrid statistical and theoretical model for the prediction of vibration levels in thin plates. The model seem to predict the effect on the vibration levels from changing thickness, density and stiffness of the plates in a heuristic correct manner. The RLM needs more validation and calibration, especially for a better prediction of frequencies of signature modes when the plates are very thin or very thick. The RLM predicts a weaker admittance level of the first air resonance, A0, with a thinner back plate while statistical analysis on a larger set of instruments has shown the opposite trend [27]. This should be investigated further, e.g. include the effect of radiation through the f-holes from signature modes [Bissingers article or ASA speech on near field holography over f-holes].

A mix of a statistical acoustics model for the radiation efficiency and an analytical model for the radiation efficiency and average vibration velocity of thin plates give some clue to the high frequency response of violins, or similar structures. Example the trade off between plate thickness, vibration levels and the radiated sound can be modelled, but a better model for the radiation around the critical frequency should be developed or found. The radiated response at very high frequencies seem to be exaggerated and is likely to be incorrect. A proper force function other than a flat and frequency independent function should be added. This model need more validation, especially methods to determine the radiation efficiency experimentally, or at least methods to improve the predicting power like sound speed in the length direction of the plates and measure the wood density.

The RL and FAR models are far from being perfect for predicting the violin timbre in detail. But the “black box” models is assumed to give the correct directions to what happens when you change thickness or any material parameters like: what direction the radiation efficiency will go, how the signature modes are likely to be influenced, you may even for fun predict how well you would do in the Swedish international violin competition that particular year.
More serious: With these models you should be able to determine a first guess for what compensations you should do if you choose another wood type, a composite material, or just a specific type of material. Can I use my very nicely flamed back with a low E module? How should I pair the plates to achieve a particular admittance pattern? Can I buy material that other makers does not want, and still get a good result?
The RL- and FAR models may even be used as a first guide to copy a particular violin frequency response with some care in the use of the models.

Two years of experience with these models are this far quite good, but there is need for a re-calibration when more data is collected to verify them.
A sound addition would be to include the mean value response theory developed for the higher frequency response of the violin body and bridge by Woodhouse [28]. This has a statistical nature similar to the radiation model for frequencies over say 1-1,5kHz, and includes the apparently important properties of the bridge. (Although quite simple in form I do not know how to interpret this yet)

But other important factors not yet properly determined are:
How does the bass bar influence the radiation efficiency? Does size matter? Height? Length? How important is the radiation efficiency as compared to the bridge/body hill?

Those capable of doing heavy calculations on the violin could also guide us here. Are any BEM, FEM or time domain models able to model the entire instrument properly now? If so it would have been very interesting to get sensitivity plots or numbers given along the same lines as given in this article. We need to know better the effect different changes has on the amplitudes of the plates and its influence on the radiated sound.

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